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The magic of the Fibonacci sequence and the golden ratio

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Across a multitude of creative disciplines and industries, artists, designers, musicians, and architects have all exploited this mathematical marvel in their work. Variations of this captivating number pattern, known as the Fibonacci sequence, show up all around us and have captivated the public imagination for centuries.

The Fibonacci sequence, a famous series of numbers, begins with zero and one and proceeds with terms equivalent to the sum of the previous two terms in the sequence. The sum of the first two terms (zero and one) gives the newest term in the sequence. This begins the never-ending progression. The sequence starts like this: 0 1 1 2 3 5 8 13 and continues infinitely.

This simple and seemingly arbitrary sequence is so significant that there is even a term to describe its relevance and the relationship between the natural world and itself. The 'golden ratio' was used by the ancient Greek mathematician Euclid many centuries ago and is closely related to Fibonacci numbers. It defines just how fundamental the pattern is in nature, music, art and even the built environment.

The golden ratio, in one of its many forms, ultimately defines the relationship between two numbers as the ratio of one to 1.618. Represented by the Greek letter Phi, its connection to the Fibonacci sequence is clear as the ratio of 1.618 can be approximated by dividing one Fibonacci number by the previous in the sequence. For example, the ratio between numbers three and two in the series loosely approaches Phi (1.618), so too does that between five and three, that between 13 and eight and so on.

For hundreds of years, even dating back to the Renaissance era, creatives have employed The golden ratio in their work. During the Renaissance, artists referred to it as The Divine Proportion (or De Divina Proportione as used by Leonardo da Vinci). Da Vinci made a habit of applying the Divine Proportion in his craft, including in his renowned Mona Lisa, as a tool to achieve accurate proportionality.

In one of its many configurations, the golden ratio can be used as the proportion of shorter line segments to longer line segments in the ratio one to 1.618. In The Mona Lisa, da Vinci made use of subdivisions in the painting to achieve this precision. Referred to as a 'golden rectangle', within each rectangular subdivision on the canvas existed a smaller rectangle and a perfect square. The mathematical phenomenon hidden behind the grid-like structure used in many of da Vinci's works aided him in achieving the balance and lifelike accuracy for which he is so famously known.
Not only are Fibonacci numbers inherent in art, but in music too. Even the keys on a piano are made up of these digits: An octave is comprised of 13 notes or keys, while eight of these are white keys and five are black, all of which, upon looking at the sequence, are Fibonacci numbers. (They appear in the sequence).

Today, we know that Mozart was enthusiastic about mathematics. In fact, Mozart and maths were so interconnected that the term 'Mozart Effect' was coined. The phrase refers to the idea that children who listen to Mozart's pieces will, hypothetically, increase their IQs. Not only was there a broad correlation between Mozart's proficiency in mathematics and music; he even applied the golden ratio in his compositions.

A traditional piano sonata has two major divisions: the exposition being the first, and the development and recapitulation being the second. The aforementioned part introduces the musical theme, while the latter is where the theme is extended and repeated. In Mozart's sonatas, the number of bars in these two parts is set up in such a way as to recreate the golden ratio, one to 1.618, between the development and exposition components. Here, again, is evidence of the importance of this sequence in the worlds of music, art and innovation.

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It is even said that part of the number set in the Fibonacci sequence manifests in the stock market. According to the Smithsonian Magazine, in March 2020, amid economic mayhem, market researchers looked to a tool to predict market movements. This tool, known as 'Fibonacci retracement', is used by technical analysts to inform their outlooks on market trade behaviour. The fact that this number sequence somehow aids in market predictions is just another demonstration of its gravity, even so many centuries on.

Although this notion has been disputed by some research experts, others have found definite connections between the sequence and market movements, all through the lens of the golden ratio. For example, the reciprocal of the golden ratio, one over Phi, is approached by dividing a Fibonacci number by its successor. Both 61.8 percent and 38.2 percent have been significant in these predictions, as they approximate multiples and reciprocals of Phi.

These percentages have been significant to the research done by analyst, Katie Stockton. As founder and managing partner of Fairlead Strategies, she applied Fibonacci numbers in investigating the climbing of the S&P 500 index during the pandemic.

While the presence of Fibonacci numbers in market movements is contested, there are noticeable investigations into its relevance, and it is still used as an assistant indicator in the field.

Amongst all the occurrences of, and uses for, maths in the real world, the most fascinating, perhaps, is the existence of the golden ratio in nature. It cannot be replicated or manufactured, as it simply is. The natural world constantly exhibits this mathematical phenomenon.

In a recent investigation done by Johns Hopkins University, it was found that even the dimensions of the human skull follow the golden spiral. From the number of spirals in a pinecone to the helix shape of a snail's shell and the number of petals in each layer of a sunflower, the geometry set up by the golden ratio is prolific in nature.

This may all seem hard to believe, and while some of these instances seem even less likely than others, one of the many significances behind this captivating wonder is the way in which everything falls into place. Mathematics exists all around us, in both the natural and physical worlds, and exhibits the harmony by which the world is governed. It provides us with a formula for making sense of the incomprehensible.

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